NOTE N-595(R)

# SYNTHESIS OF AERODYNAMIC FORCE CHARACTERISTICS OF BISYMMETRIC LIFTING VEHICLES

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## FOREWORD

The work reported here was originally done under the T-16 Subtask, Comparison of Lifting and Ballistic Reentry Spacecraft for Military Purposes. Results of the initial application of the derived expressions to evaluate glide and landing characteristics of low L/D vehicles are described in IDA Study S-112 (Aug 1963). The derived linearized expressions have been incorporated in the RANGE program and are used in all aircraft landing, lifting-reentry, and horizontal-takeoff launch vehicle trajectory calculations; the linear approximations are fundamental to all other RANGE trajectories involving lift. Subsequent IDA documents whose results make use of these routines include P-237 (Jan 1966), R-114 (Feb 1966), P-427 (May 1968), and P-425 (Jun 1969). This present document is assembled to provide background detail for these aerodynamic subroutines which are in continuing use in RANGE.

## ACKNOWLEDGMENT

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## ABSTRACT

The dependences of lift and drag coefficients on angle of attack for bisymmetric lifting vehicles are synthesized using two simplifying assumptions:

1. the axial force coefficient is independent of the angle of attack, and
2. the normal force coefficient increases linearly with the angle of attack.

Good agreement with experimental data is found for the value of the angle of attack at which the maximum L/D occurs and the angle of attack for maximum $C_L$ in the hypersonic regime; applicability for subsonic flight is limited to those angles of attack below the inception of flow separation.

Conditions for terminal equilibrium glide and landing for aircraft are derived from the simplified aerodynamic characteristics.

The following useful rules of thumb are obtained for super and hypersonic vehicles with maximum L/D greater than 1.0:

1. the drag at maximum L/D is very closely twice the zero-lift drag,
2. the L/D at maximum $C_L$ is about 0.8,
3. the angle of attack for maximum $C_L$ is about 48 deg,
4. the ratio of the maximum $C_L$ to the $C_L$ at $L/D_{max}$ is approximated by $0.5 + L/D_{max}$, and
5. the ratio of the normal force slope to the maximum lift is closely 2.0 per radian.

# NOMENCLATURE

| | |
|---|---|
| $A$ | Reference area |
| $C_A$ | Axial force coefficient |
| $C_D$ | Drag coefficient |
| $C_L$ | Lift coefficient |
| $C_N$ | Normal force coefficient |
| $C_{N_\alpha}$ | Factor relating $C_N$ and its $\alpha$ dependence |
| $C_V$ | Vertical force coefficient = vertical force/$\frac{1}{2}\rho V^2 A$ |
| $D$ | Drag force |
| $f$ | Ratio of vertical force to effective weight |
| $F_A$ | Axial force |
| $g_{flare}$ | Flare acceleration in g's |
| $L$ | Lift force |
| $V$ | Velocity |
| $W_{eff}$ | Effective weight = mass times local acceleration of gravity less the centrifugal force due to velocity |
| $\alpha$ | Angle of attack |
| $\alpha_o$ | Initial angle of attack |
| $\alpha_1$ | Angle of attack at which maximum L/D occurs |
| $\alpha_2$ | Angle of attack at which maximum vertical force (or $C_L$) occurs |
| $\alpha_3$ | Angle of attack at which maximum or minimum $C_N$ occurs |
| $\gamma$ | Flight path angle with horizontal, positive upwards |
| $\gamma_o$ | Initial or desired flight path angle |
| $\rho$ | Atmospheric density |

CONTENTS

# I. INTRODUCTION

Calculation of the flight path of a point mass vehicle acting under the influence of drag and lift forces requires, in addition to the more commonly available dependences of these forces on Mach and Reynolds numbers, the dependences of the lift-drag ratio and lift or drag on the angle of attack. (With a point mass vehicle the drag and lift are assumed to act through the center of mass; the otherwise interesting moments are assumed to have been cancelled out by hypothetical control surfaces.) A description is given here of the synthesis of these aerodynamic force characteristics for a vehicle configuration simplified but sufficiently general to generate flight behavior information indicative of the performance of most real vehicles in the angle of attack range 0-20 deg for subsonic flight and 0-50 deg for hypersonic flight.

The subject vehicle is required to have two basic limitations. The first is that it should have mirror symmetry in the two perpendicular planes, vertical and horizontal, whose intersection forms the longitudinal axis of the vehicle. Its second limitation, really an extension of the symmetry limitation, is that stability and attitude control are to be supplied by control surfaces or forces giving negligible drag perturbation or mass expenditure. In other words it should be a bisymmetrical body-wing configuration with symmetry not

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only left to right but also with symmetry in the top and bottom surfaces (airfoil) and having a method of attitude control that does not disturb the symmetry. Bodies of revolution, e.g., cylindrical or conical missile bodies, are included in the family of vehicles covered by these limitations. The symmetry limitation is to assure that the force normal to the vehicle is zero for zero angle of incidence between the longitudinal axis and the flight path.

## II. BASIC ASSUMPTIONS

Two assumptions about the aerodynamic force characteristics of the subject vehicle are required, from which the further necessary relations are easily derivable. The two basic assumptions for this paper are determined empirically from inspection of a wide range of experimental results. The first is that the axial force coefficient, the relative aerodynamic force in the direction of the vehicle's longitudinal axis, is a constant independent of the angle of attack. The second is that the normal force coefficient, the relative aerodynamic force perpendicular to the longitudinal axis, has a quadratic dependence on the angle of attack. In equation form these assumptions are

$$C_A = \text{constant} \tag{1}$$

$$C_N = C_{N_\alpha} \alpha(1 - \alpha/2\alpha_m) \tag{2}$$

where $\alpha_m$ is the angle at which a maximum or minimum value of $C_N$ occurs.

The first assumption is good, i.e., within 20% in the first $20^\circ$ of $\alpha$, for thick delta wings (Ref. 1, Mach 3-6), excellent, i.e., within 10% for $\alpha < 20^\circ$, for cylindrical bodies (Ref. 2, Mach 6.86), but of varying applicability for sharp cones which show either a 20% drop in $C_A$ (Ref. 2, Mach 2) or a 50% increase in $C_A$ (Ref. 5, Mach 6.77) in the first $12^\circ$ of $\alpha$. The accuracy of the assumption becomes poorer as the Mach number decreases below 2, but remains within 50% at $\alpha < 12^\circ$ even at low subsonic speeds (Ref. 4).

The quadratic $C_N$ dependence on $\alpha$ could be made to follow the data within a few percent up to $\alpha \approx 45^\circ$ throughout the whole range of Mach numbers if $\alpha_s$ were allowed to vary with M. Generally $\alpha_s$ would have a large negative value leading to a $C_N$ vs $\alpha$ curve that is slightly concave upward (Refs. 1, 2, and 5). The value of $\alpha_s$ would become closer to zero as the Mach number increases, leading to a more nonlinear $C_N$ vs $\alpha$ curve.

A linear approximation to assumption 2 ($\alpha_s = \pm \infty$) is generally acceptable (deviation less than 10%) up to $\alpha = 20^\circ$ and sometimes higher. Newtonian impact theory predicts an increase in $C_D$ with $\alpha$ (Ref. 10),

$$\frac{\partial C_D}{\partial \alpha} = 3\ C_L$$

that is as much as 30 to 40% greater than supported by experiment (Refs. 11 and 12) and about 50% greater than that given by applying the assumption $C_A$ = constant to the differential of equation 4 below.

## III. DERIVATION OF RELATIONS

The basic relationship is the transformation from the body reference system of forces, i.e., the axial and normal forces on the vehicle, to the flight-path reference system defined by the lift force perpendicular to the flight path and the drag force parallel with the flight path. From the force diagram in Figure 1

$$C_L = C_N \cos\alpha - C_A \sin\alpha \tag{3}$$

$$C_D = C_N \sin\alpha + C_A \cos\alpha \tag{4}$$

$$\text{or} \quad C_L/C_D = \tan\left(\tan^{-1} C_N/C_A - \alpha\right) \tag{5}$$

The reverse transformation is

$$C_N = C_L \cos\alpha + C_D \sin\alpha \tag{6}$$

$$C_A = C_D \cos\alpha - C_L \sin\alpha \tag{7}$$

$$\text{or} \quad C_N/C_A = \tan\left(\tan^{-1} C_L/C_D + \alpha\right) \tag{8}$$

Differentiation of equation 5 with respect to $\alpha$ (with substitution of equation 2) and setting the result equal to zero gives a preliminary relation for $\alpha_1$ at which maximum L/D occurs

$$\alpha_1 = \frac{\sqrt{(C_{N_\alpha}/C_A)\,(1 - \alpha_1/\alpha_m) - 1}}{(C_{N_\alpha}/C_A)\,(1 - \alpha_1/2\alpha_m)} \tag{9}$$

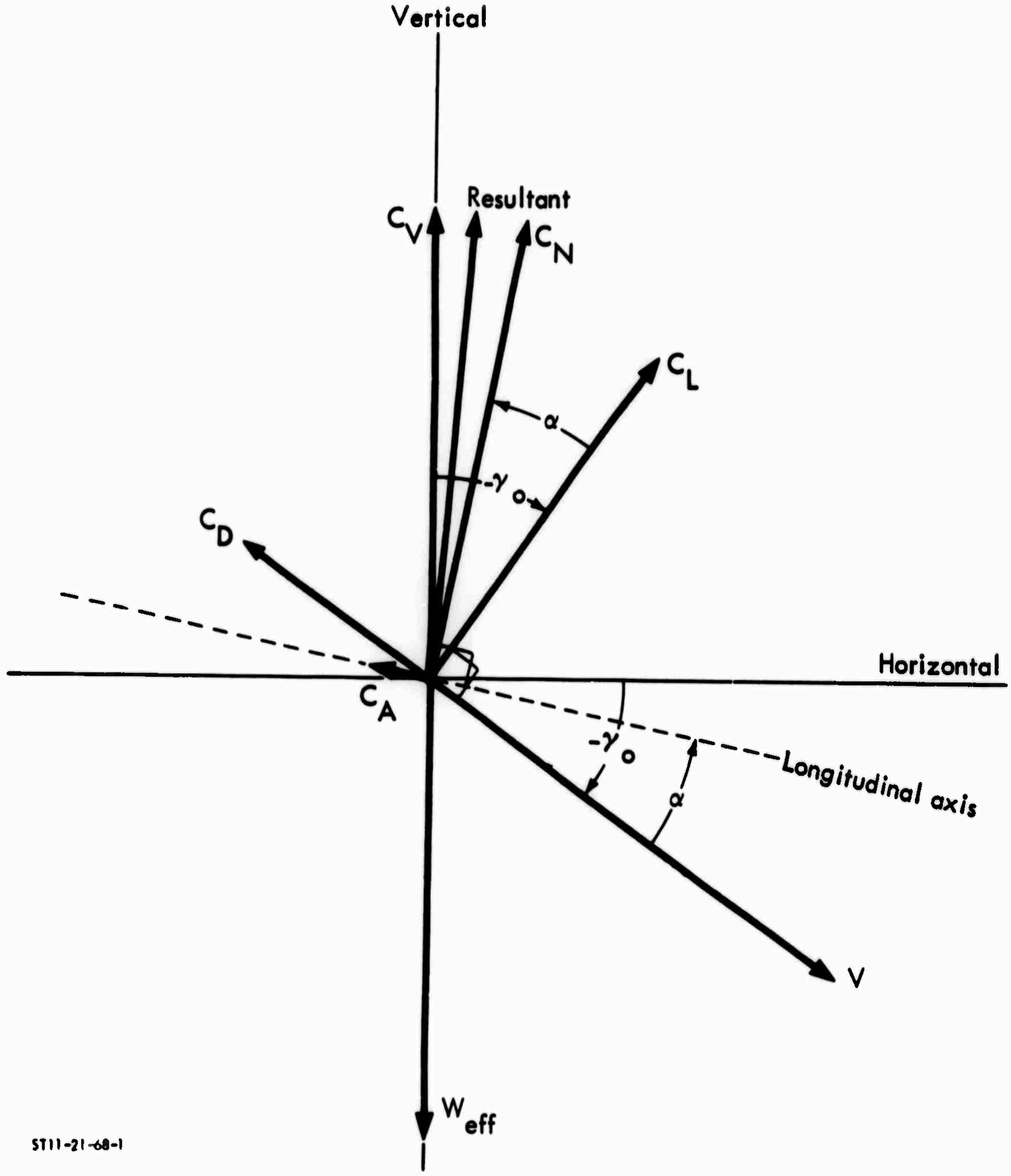


FIGURE 1. Force Diagram for a Gliding Aircraft Showing Relationship of Flight-Path-Oriented Force Coefficients, $C_L$ and $C_D$, with Body-Oriented Force Coefficients, $C_N$ and $C_A$, and with an Earth-Oriented Force Coefficient, $C_V$

or

$$(C_{N_\alpha}/C_A)(1 - \alpha_1/\alpha_s) - 1 = (C_N/C_A)^2\Big|_{\alpha = \alpha_1} \tag{10}$$

A most useful defining parameter of the vehicle is the maximum L/D value. Substituting equation 8 in equation 10, we obtain as an intermediate result in terms of $L/D_{max}$

$$C_{N_\alpha}/C_A = \frac{\sec^2 (\tan^{-1} L/D_{max} + \alpha_1)}{(1 - \alpha_1/\alpha_s)} \tag{11}$$

which, when substituted in equation 9, gives

$$\alpha_1 = \frac{\tan (\tan^{-1} L/D_{max} + \alpha_1)}{\sec^2 (\tan^{-1} L/D_{max} + \alpha_1)} \frac{(1 - \alpha_1/\alpha_s)}{(1 - \alpha_1/2\alpha_s)} \tag{12}$$

which can be finally simplified to

$$\alpha_1 = \tfrac{1}{2} \sin 2 (\tan^{-1} L/D_{max} + \alpha_1) \frac{(1 - \alpha_1/\alpha_s)}{(1 - \alpha_1/2\alpha_s)} \tag{13}$$

Using this value of $\alpha_1$ and a combination of equations 2 and 8, a slightly simpler relation than equation 11 is obtained

$$C_{N_\alpha}/C_A = \frac{\tan (\tan^{-1} L/D_{max} + \alpha_1)}{\alpha_1 (1 - \alpha_1/2\alpha_s)} \tag{14}$$

Differentiating equation 3 with respect to $\alpha$ (with substitution of eqs. 1 & 2) and setting equal to zero gives a relation for $\alpha_2$ at which maximum $C_L$ occurs

$$\alpha_2 \tan \alpha_2 = \frac{C_{N_\alpha}/C_A (1 - \alpha_2/\alpha_s) - 1}{C_{N_\alpha}/C_A ( 1 - \alpha_2/2\alpha_s)} \tag{15}$$

For glide at an angle $\gamma_o$ negative below the horizontal, the vertical force coefficient (in an earth-oriented system rather than body-oriented or flight-path-oriented, Fig. 1) is

$$C_V = C_N \cos(\gamma_o + \alpha) - C_A \sin(\gamma_o + \alpha) \qquad (16)$$

Maximizing this with respect to $\alpha$ gives an alternative equation for $\alpha_s$, of which 15 is a special case for $\gamma_o = 0$,

$$\alpha_s \tan(\gamma_o + \alpha_s) = \frac{C_{N_\alpha}/C_A\ (1 - \alpha_s/\alpha_m) - 1}{C_{N_\alpha}/C_A\ (1 - \alpha_s/2\alpha_m)} \qquad (17)$$

This gives the angle of attack at which the maximum vertical force is available to act against the vehicle weight.

One additional relationship is obtained from inspection of equation 8. If $\tan^{-1}(L/D + \alpha)$ is greater than $\pi/2$, then $C_N/C_A$ is negative, which is illogical. Consequently, we have a constraint on the logical values of $\alpha$ for a given value of L/D,

$$\alpha \leq \pi/2 - \tan^{-1} L/D \qquad (18)$$

The region of $\alpha$ greater than this quantity is an excluded region.

Figure 2 shows a set of plots of relations 5, 13, and 18 describing the dependences of L/D on $\alpha$ for vehicles with $\alpha_m = \infty$ and different values of $L/D_{max}$, the locus of the maxima of L/D vs $\alpha$ curves for vehicles with $\alpha_m = \infty$, and finally the boundary of the excluded region. Figure 3 gives the equivalent dependences to L/D on $C_L$ from equations 3 and 15, with $C_L$ defined in terms of the ratio to $C_{L_{max}}$.

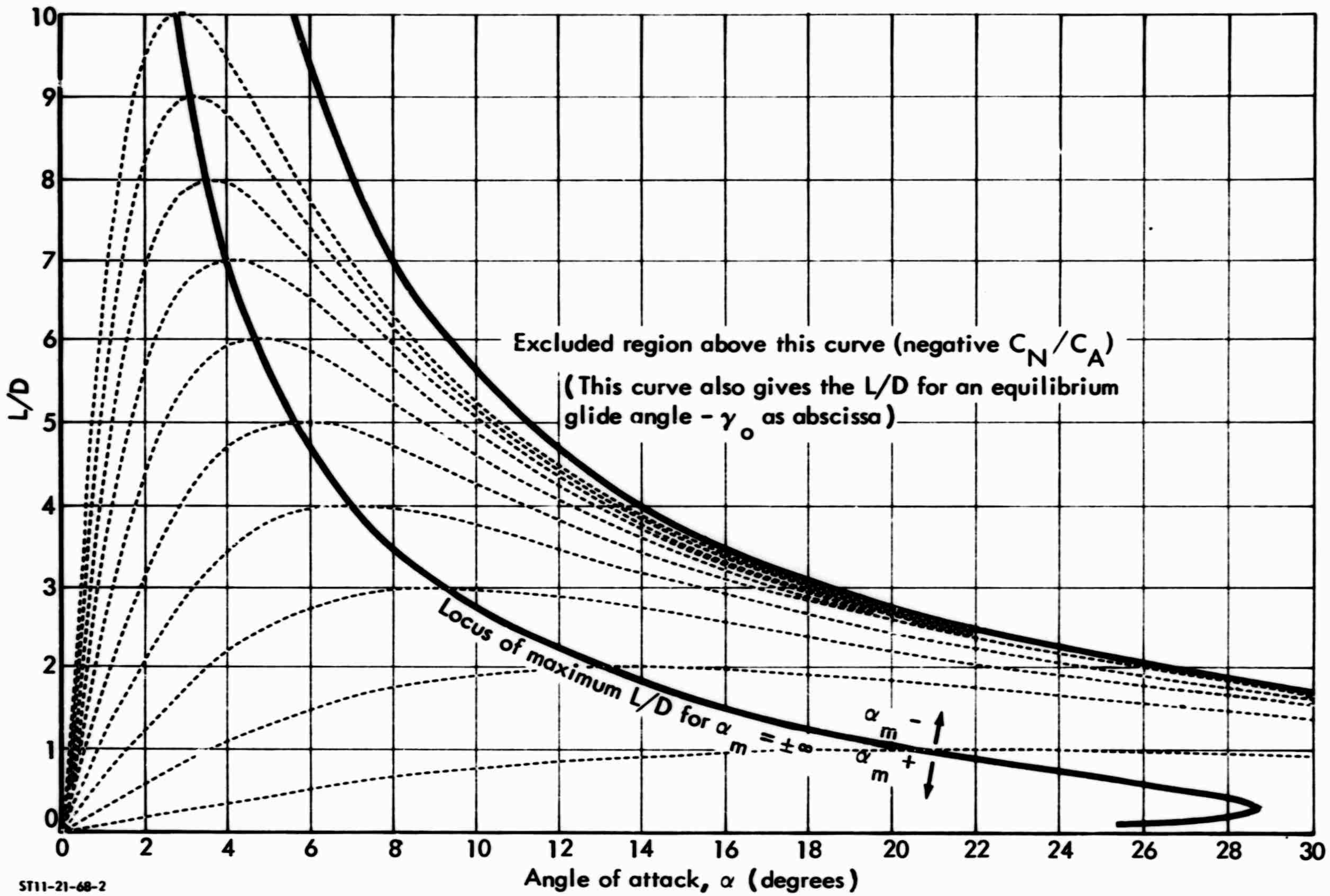


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FIGURE 2. Basic L/D Dependences: L/D Versus Angle of Attack for Vehicles with Different Values of $L/D_{max}$ and $\alpha_m = \infty$; Angle of Attack for Maximum L/D Versus $L/D_{max}$; and L/D Required for an Equilibrium Glide Angle $\gamma_o$

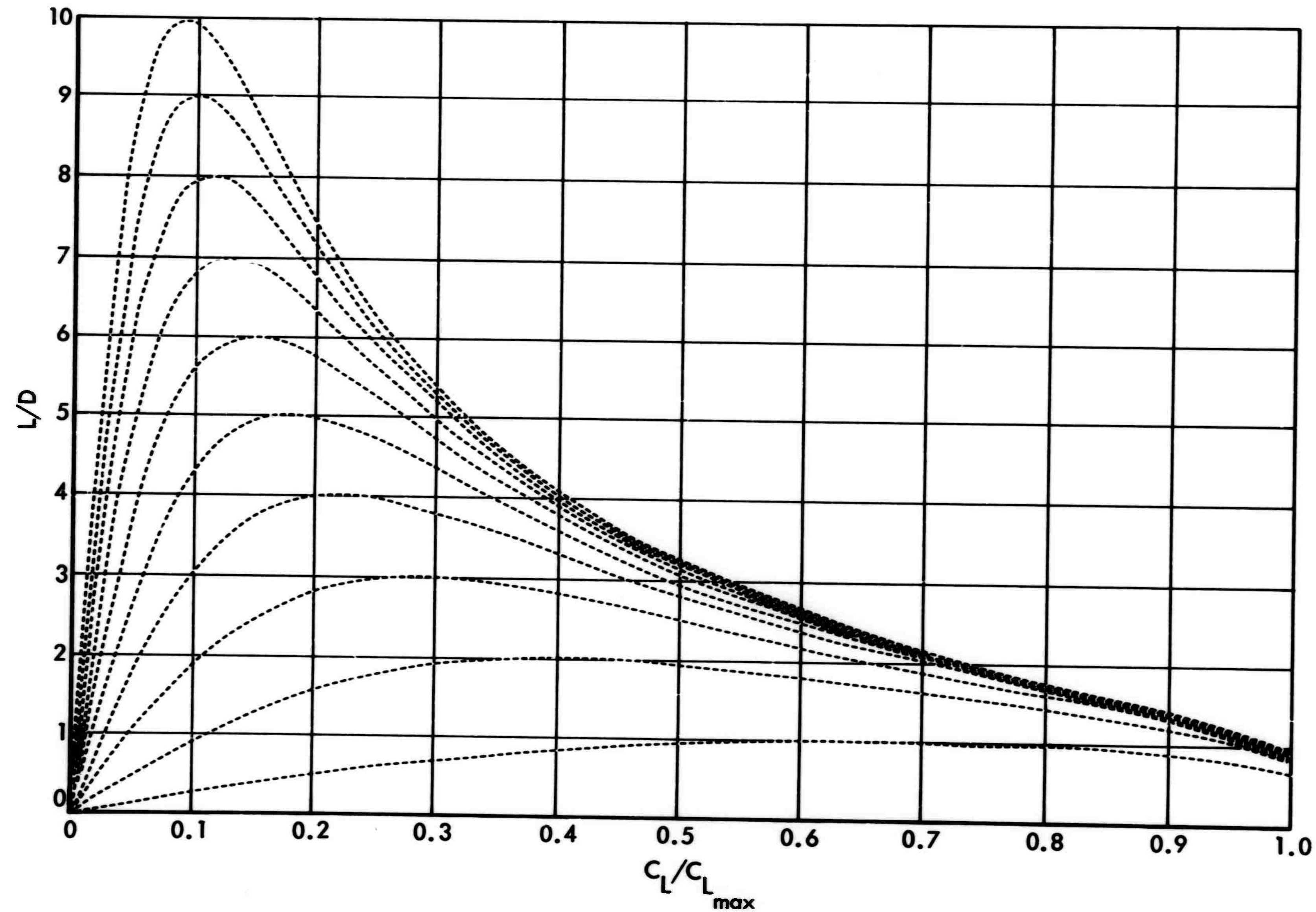


FIGURE 3. L/D Versus $C_L$ ( in Terms of $C_{L_{max}}$ ) for Vehicles with Different Values of $L/D_{max}$ and $\alpha_m = \infty$

## IV. EQUILIBRIUM GLIDE

A particular vehicle requires the value of one more parameter in addition to $L/D_{max}$ to define completely its aerodynamic force characteristics for a point-mass trajectory calculation. That parameter will most likely be its axial force coefficient, i.e., its zero-lift drag coefficient, for subsonic applications or its maximum lift coefficient for hypersonic applications. For a vehicle in subsonic glide, the value of $C_A$ can be obtained, for example, from the vehicle's terminal velocity at a constant glide angle (equilibrium glide condition), in the following manner:

For an equilibrium glide at an angle $\gamma_o$ which is measured from the horizontal and negative downward, the resultant of the aerodynamic forces in Fig. 1 is equal and directly opposite to the effective weight $W_{eff}$, i.e.,

$$L = W_{eff} \cos \gamma_o \tag{19}$$

$$D = -W_{eff} \sin \gamma_o \tag{20}$$

or

$$L/D = -\cot \gamma_o \tag{21}$$

This relationship is the same as the equality in 18 with $\alpha$ replaced by $-\gamma_o$. With $L/D_{max}$ in place of L/D, this expression also gives the minimum equilibrium glide angle. The boundary of the excluded region in Figure 1 therefore gives the L/D required for equilibrium glide at an angle $\gamma_o$ reading $-\gamma_o$ as the abscissa instead of $\alpha$, or the minimum glide angle, $\gamma_{min}$, for vehicles with different $L/D_{max}$.

From equation 21 and equation 8 we can derive the angle of attack of the vehicle in this equilibrium glide

$$\alpha_o = -\gamma_o - \cot^{-1}\left[(C_{N_\alpha}/C_A)\,\alpha_o\,\left(1 - \frac{\alpha_o}{2\alpha_m}\right)\right] \tag{22}$$

where the solution $\alpha_o < \alpha_1$ is chosen. Finally from equation 7 and equations 19 and 20

$$C_A = C_D \cos\alpha_o - C_L \sin\alpha_o$$

$$= -\frac{W_{eff}\sin\gamma_o}{1/2\rho V^2 A}\cos\alpha_o - \frac{W_{eff}\cos\gamma_o}{1/2\rho V^2 A}\sin\alpha_o$$

$$= \frac{W_{eff}}{1/2\rho V^2 A}\sin(-\gamma_o - \alpha_o) \tag{23}$$

If $L/D_{max}$ and $C_A$ are given, then the equilibrium glide velocity is found from equation 23,

$$V = \left[\frac{2}{\rho}\left(\frac{W_{eff}}{C_A A}\right)\sin(-\gamma_o - \alpha_o)\right]^{1/2}$$

## V. RECAPITULATION FOR LINEAR DEPENDENCE OF $C_N$ ON $\alpha$

In subsonic glide situations, only angles of attack less than $20^{\circ}$ are of general interest. In this region of values of $\alpha$ a linear dependence of $C_N$ on $\alpha$ may be used with little error, giving some simplification to the equations. The simplified equations are listed below in the order of their calculation with a description of their method of solution. The corresponding FORTRAN subroutines are listed in Appendix A.

The angle of attack for $L/D_{max}$

$$\alpha_1 = \frac{1}{2} \sin 2 \left(\tan^{-1} L/D_{max} + \alpha_1\right) \qquad (13')$$

is a transcendental equation. It is solved by iteration from a first guess that is very close

$$\tilde{\alpha}_1 = \frac{1}{2} \left(\pi/2 - \tan^{-1} L/D_{max}\right) = - \frac{1}{2} \gamma_{min} \qquad (24)$$

Each iteration input after the first is the average of the input and output of the previous iteration.

The ratio of the normal-force-coefficient slope to the axial-force coefficient is a simple direct equation

$$C_{N_\alpha}/C_A = \frac{\tan\left(\tan^{-1} L/D_{max} + \alpha_1\right)}{\alpha_1} \qquad (14')$$

The angle of attack for maximum vertical force (maximum $C_L$ for horizontal flight)

$$\alpha_s = \frac{C_{N_\alpha}/C_A - 1}{(C_{N_\alpha}/C_A)\tan(\gamma_0 + \alpha_s)} \qquad (17')$$

is another transcendental equation to be solved by iteration. The first guess is

$$\tilde{\alpha}_s = 0.86 - 0.64\ \gamma_0 \qquad (25)$$

Each iteration input after the first is a value only one-quarter of the way from the input to the output of the previous iteration, to guarantee convergence for an otherwise divergent process.

With the above equations as programmed in the subroutines in Appendix A, values are calculated for $\alpha_1$, $\alpha_s$, $C_{N_\alpha}/C_A$, $C_{L_1}/C_A$, $C_{L_{max}}/C_A$, $C_{L_{max}}/C_{D_s}$, and $\gamma_{min}$, and are listed in Table 1 for the interesting range of values of $L/D_{max}$. The ratios $C_{L_{max}}/C_{L_1}$, $C_{N_\alpha}/C_{L_{max}}$, $C_{D_1}/C_A$, and $C_{L_{max}}/C_{D_s}$ are plotted in Fig. 4. The values in the Figure are more limited in their range than the values in the table, and indeed the latter three of them are essentially constant for $L/D_{max}$ values greater than 1.0.

With the linear $C_N$ dependence, the angle of attack for equilibrium glide becomes

$$\alpha_0 = -\gamma_0 - \pi/2 + \tan^{-1}((C_{N_\alpha}/C_A)\ \alpha_0) \qquad (22')$$

The solution to this equation can best be shown graphically with the aid of Figure 2. Reading up from $-\gamma_0$, as the abscissa, to the boundary curve gives the required L/D. Reading across with this L/D there are in general two intersections, or none, with an L/D vs $\alpha$ dashed curve.

TABLE I

Aerodynamic Force Characteristics of Generalized Vehicles with Different $L/D_{max}$

| $L/D_{max}$ | $\alpha_1$ | $\alpha_2$ | $C_{N_\alpha}/C_A$ | $C_{L_1}/C_A$ | $C_{L_{max}}/C_A$ | $C_{L_{max}}/C_{D_0}$ | $-\gamma_{min}$ |
|---|---|---|---|---|---|---|---|
| .100 | 25.307 | 28.274 | 1.361 | .116 | .118 | .096 | |
| .200 | 28.101 | 34.108 | 1.675 | .254 | .265 | .191 | |
| .300 | 28.643 | 37.621 | 2.024 | .409 | .442 | .276 | |
| .400 | 28.209 | 40.035 | 2.421 | .578 | .652 | .352 | |
| .500 | 27.289 | 41.796 | 2.874 | .758 | .897 | .418 | 63.43 |
| .600 | 26.132 | 43.124 | 3.389 | .947 | 1.178 | .476 | |
| .700 | 24.875 | 44.150 | 3.968 | 1.142 | 1.497 | .526 | |
| .800 | 23.602 | 44.955 | 4.616 | 1.342 | 1.857 | .568 | |
| .900 | 22.359 | 45.597 | 5.336 | 1.545 | 2.257 | .604 | |
| 1.000 | 21.173 | 46.116 | 6.128 | 1.750 | 2.698 | .635 | 45.00 |
| 1.100 | 20.056 | 46.538 | 6.994 | 1.957 | 3.182 | .661 | |
| 1.200 | 19.014 | 46.885 | 7.936 | 2.164 | 3.709 | .684 | |
| 1.300 | 18.046 | 47.175 | 8.955 | 2.372 | 4.278 | .703 | |
| 1.400 | 17.150 | 47.417 | 10.050 | 2.580 | 4.892 | .719 | |
| 1.500 | 16.323 | 47.622 | 11.223 | 2.787 | 5.549 | .733 | 33.69 |
| 1.600 | 15.558 | 47.796 | 12.475 | 2.995 | 6.250 | .746 | |
| 1.700 | 14.852 | 47.945 | 13.804 | 3.202 | 6.995 | .756 | |
| 1.800 | 14.199 | 48.074 | 15.213 | 3.409 | 7.785 | .766 | |
| 1.900 | 13.594 | 48.186 | 16.700 | 3.616 | 8.619 | .774 | |
| 2.000 | 13.034 | 48.283 | 18.267 | 3.823 | 9.497 | .781 | 26.55 |
| 2.100 | 12.513 | 48.369 | 19.912 | 4.029 | 10.420 | .788 | |
| 2.200 | 12.029 | 48.444 | 21.637 | 4.235 | 11.387 | .793 | |
| 2.300 | 11.579 | 48.511 | 23.441 | 4.440 | 12.399 | .798 | |
| 2.400 | 11.158 | 48.570 | 25.325 | 4.645 | 13.456 | .803 | |
| 2.500 | 10.765 | 48.623 | 27.289 | 4.850 | 14.557 | .807 | 21.82 |
| 2.600 | 10.397 | 48.671 | 29.332 | 5.055 | 15.704 | .811 | |
| 2.700 | 10.052 | 48.714 | 31.455 | 5.259 | 16.894 | .814 | |
| 2.800 | 9.728 | 48.752 | 33.657 | 5.464 | 18.130 | .817 | |
| 2.900 | 9.423 | 48.787 | 35.940 | 5.667 | 19.410 | .820 | |
| 3.000 | 9.136 | 48.819 | 38.302 | 5.871 | 20.736 | .822 | 18.43 |
| 3.100 | 8.865 | 48.847 | 40.744 | 6.075 | 22.106 | .824 | |
| 3.200 | 8.609 | 48.874 | 43.265 | 6.278 | 23.520 | .827 | |
| 3.300 | 8.367 | 48.898 | 45.867 | 6.481 | 24.980 | .828 | |
| 3.400 | 8.138 | 48.920 | 48.548 | 6.685 | 26.484 | .830 | |
| 3.500 | 7.920 | 48.940 | 51.310 | 6.887 | 28.034 | .832 | 15.94 |
| 3.600 | 7.714 | 48.959 | 54.151 | 7.090 | 29.628 | .833 | |
| 3.700 | 7.517 | 48.976 | 57.072 | 7.293 | 31.267 | .835 | |
| 3.800 | 7.331 | 48.992 | 60.073 | 7.495 | 32.951 | .836 | |
| 3.900 | 7.152 | 49.007 | 63.154 | 7.698 | 34.679 | .837 | |
| 4.000 | 6.983 | 49.021 | 66.315 | 7.900 | 36.453 | .838 | 14.04 |
| 4.500 | 6.239 | 49.077 | 83.319 | 8.911 | 45.993 | .843 | 12.52 |
| 5.000 | 5.636 | 49.117 | 102.322 | 9.919 | 56.655 | .846 | 11.31 |
| 5.500 | 5.138 | 49.147 | 123.324 | 10.926 | 68.439 | .848 | 10.30 |
| 6.000 | 4.720 | 49.170 | 146.325 | 11.932 | 81.345 | .850 | 9.46 |
| 6.500 | 4.365 | 49.188 | 171.326 | 12.937 | 95.373 | .852 | 8.75 |
| 7.000 | 4.058 | 49.203 | 198.327 | 13.941 | 110.523 | .853 | 8.13 |
| 7.500 | 3.792 | 49.215 | 227.328 | 14.945 | 126.795 | .854 | 7.59 |
| 8.000 | 3.558 | 49.224 | 258.329 | 15.948 | 144.190 | .855 | 7.13 |
| 8.500 | 3.351 | 49.232 | 291.329 | 16.951 | 162.706 | .855 | 6.71 |
| 9.000 | 3.167 | 49.239 | 326.330 | 17.954 | 182.345 | .856 | 6.34 |
| 9.500 | 3.002 | 49.244 | 363.330 | 18.956 | 203.105 | .856 | 6.01 |
| 10.000 | 2.853 | 49.249 | 402.331 | 19.959 | 224.988 | .857 | 5.70 |

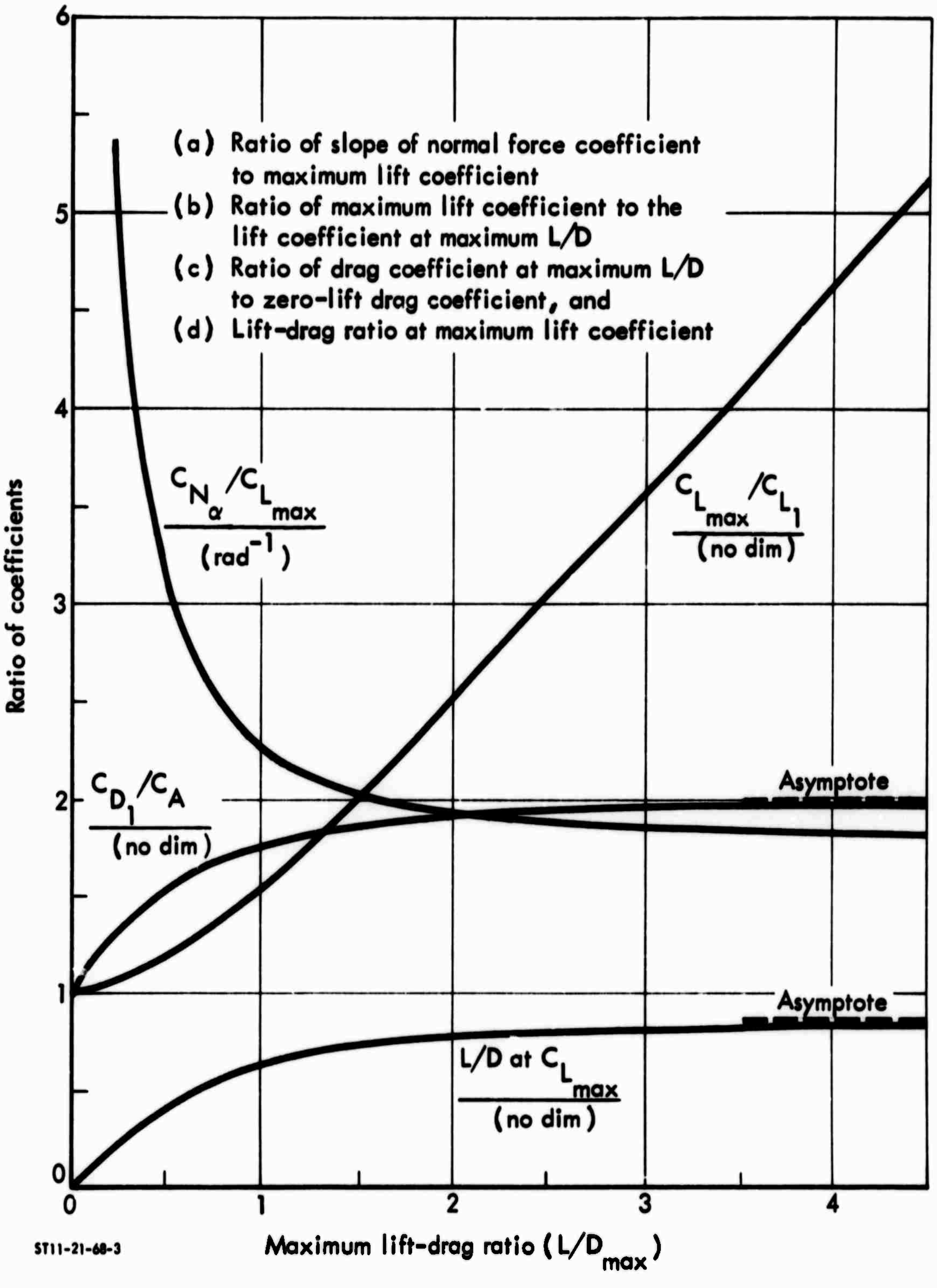


FIGURE 4. Linearized Aerodynamic Characteristics of Low L/D Aircraft

The left, lower $\alpha$, intersection is of greater interest because it represents an aircraft with a greater velocity excess above stall. Unfortunately, a simple iterative process converges on the other intersection. Consequently, equation 22' is solved by trial and linear interpolation with the first two trial values $1/2\ \alpha_1$ and $1/4\ \alpha_1$.

The axial force coefficient, or zero-lift drag coefficient, is given by equation 23 without change. Since the atmospheric density enters in equation 23, the above set of equations for equilibrium glide will give different values at different altitudes.

In subsequent flight to an altitude with a different atmospheric density, the vehicle will require an angle of attack to maintain zero vertical acceleration (which is not necessarily the same as equilibrium glide as defined here by equations 19 and 20) given by solution of the following equation by trial and linear interpolation

$$(C_{N_\alpha}/C_A)\ \alpha\ F_A\ \cos\ (\gamma + \alpha) - F_A\ \sin\ (\gamma + \alpha) - W_{eff} = 0 \qquad (26)$$

The first guess for $\alpha$ is

$$\tilde{\alpha} = W_{eff}/((C_{N_\alpha}/C_A)\ F_A) \qquad (27)$$

If a given glide slope $\gamma_o$ is to be maintained instead of zero vertical acceleration, the effective weight, $W_{eff}$, is replaced in equations 26 and 27 by $fW_{eff}$ where f has a value near 1 given by

$$f \sim 1 - (\gamma - \gamma_o) \qquad (28)$$

For a flare maneuver requiring a pullup acceleration $g_{flare}$, the expression

$$f = 1 + g_{flare} \qquad (29)$$

gives very closely the proper flare acceleration.

For horizontal flight after a flare, equation 28 is used with $\gamma_o = 0$.

Using the above relations, the performance in the glide, flare, and landing regimes has been calculated with the IDA program RANGE for general bisymmetric lifting vehicles with different wing loading and $L/D_{max}$. The results have been reported in Ref. 6.

## VI. COMPARISON OF LINEARIZED AERODYNAMIC CHARACTERISTICS WITH EXPERIMENTAL DATA

Wind-tunnel tests of some bisymmetric thick delta-wing models at Mach 3, 4.5, and 6, are reported in Ref 1. Equation 13' predicts the angle of attack $\alpha_1$ giving $L/D_{max}$ to about $1^\circ$ accuracy out of $15^\circ$. Equations 17' and 14' predict the $C_{L_{max}}$ angle of attack $\alpha_2$ about 10 percent low for the most non-linear $C_N$ vs $\alpha$ curves.

The X-20 Dyna-Soar vehicle could hardly be called symmetrical above and below the horizontal plane but it is still interesting to compare the calculations (linearized equations 13', 14', and 17') with its behavior measured in wind-tunnel tests (Refs. 4 and 7). The table below gives the comparison. The angles of attack are measured above the zero-lift attitude.

| | Subsonic ($L/D_{max}$ = 4) | | Hypersonic ($L/D_{max}$ = 2) | |
|---|---|---|---|---|
| | $\alpha_1$ | $\alpha_2$ | $\alpha_1$ | $\alpha_2$ |
| Measured | $8^\circ$ | $40^\circ$ | $12^\circ$ | $46^\circ$ |
| Calculated | $7^\circ$ | $49^\circ$ | $13^\circ$ | $48^\circ$ |

The relatively poor subsonic agreement in $\alpha_2$ is a result of flow separation.

Lift/drag data are plotted in Fig. 5 for the above vehicles and for three others superimposed on the linearized L/D vs $\alpha$ curves from Fig. 2. The curves reproduce the dependences shown by these data generally within 2 deg of angle of attack.

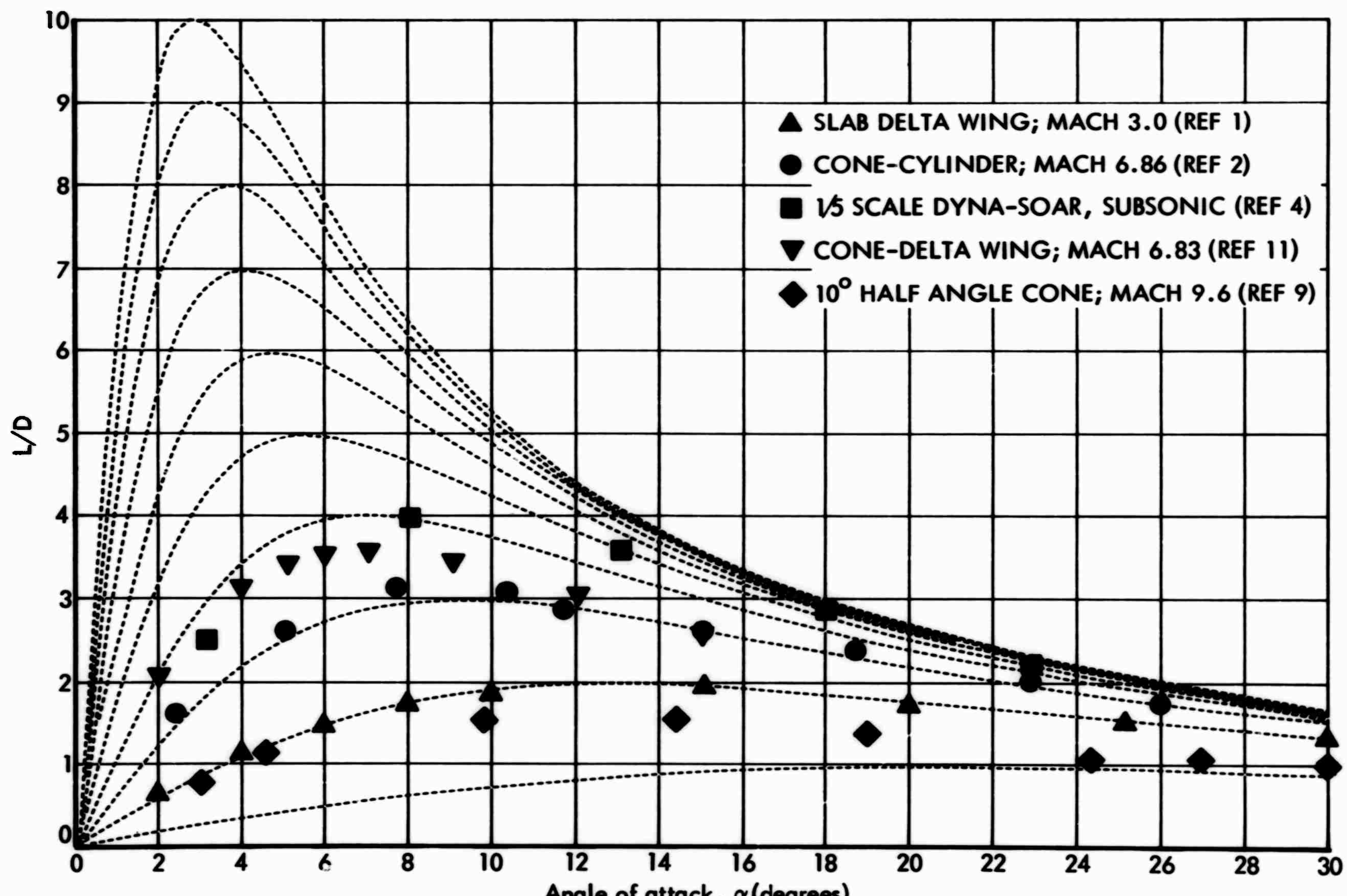


FIGURE 5. Comparison of Linearized Aerodynamic Characteristics with Experimental Data

## VII. CONCLUSIONS

Useful relations for estimating the aerodynamic force characteristics of general bisymmetric lifting vehicles in all regimes of flight can be derived from the assumptions of a linear variation of the normal force coefficient with angle of attack and an independence of the axial force coefficient on the angle of attack. Good agreement with wind-tunnel measurements is observed.

Some extremely simple rules of thumb are found from the linearized aerodynamic characteristics relations. In the range of maximum L/D above 1.0 the following approximate rules hold:

1. The drag at the angle of attack for maximum L/D is very nearly twice the zero-lift drag, or

$$C_{D_1}/C_A \approx 2$$

and

$$C_{L_1}/C_A \approx 2\ L/D_{max}$$

2. The lift-drag ratio at maximum lift coefficient is about 0.8 (cf. Ref. 8), or L/D @ $C_{L_{max}} \approx 0.8$

3. The angle of attack for maximum lift coefficient is about 48 deg, or

$$\alpha_a \approx 48 \text{ deg}$$

4. The ratio of the maximum lift coefficient to the lift coefficient at maximum L/D is given by

$$C_{L_{max}}/C_{L_1} \approx 0.5 + L/D_{max}$$

and from the first rule above

$$C_{L_{max}}/C_A \approx L/D_{max} + 2\,(L/D_{max})^2$$

5. The ratio of the normal force slope to the maximum lift decreases only from 2.25 per radian at $L/D_{max}$ = 1.0 to 1.80 per radian at $L/D_{max}$ = 5.0, so

$$C_{N_\alpha}/C_{L_{max}} \approx 2\ \text{rad}^{-1}$$

For lifting bodies, the hypersonic maximum lift coefficient is about 0.6 (Refs. 1, 7, and 8), referenced to the planform area, so

$C_{N_\alpha} \approx 1.2\ \text{rad}^{-1}$ for lifting bodies.

For cones, the hypersonic maximum lift coefficient is about 1.0 (Refs. 2 and 5), referenced to the base area, so

$C_{N_\alpha} \approx 2.0\ \text{rad}^{-1}$ for cones.

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## APPENDIX A

Calculation of Aerodynamic Force Characteristics Using Equations 13', 14', 24', 17', and 25'; and Program to Generate Table I.

```
      RADANG = .0174532925
      DO 7 LDMAX = 1,40
      ALPH1 = 0
      CNACA = 0
      FLDMAX = LDMAX
      FLDMAX = 0.1 * FLDMAX
      CALL FLT(FLDMAX, ALPH1, CNACA)
      ALPH2 = ALP(CNACA,ZERO)
      CL1CA = CNACA * ALPH1 * COSF(ALPH1) - SINF(ALPH1)
      CLMCA = CNACA * ALPH2 * COSF(ALPH2) - SINF(ALPH2)
      CLMCD = TANF(ATANF(CNACA * ALPH2) - ALPH2)
      ALPH1D = ALPH1/RADANG
      ALPH2D = ALPH2/RADANG
    7 PRINT 9, FLDMAX, ALPH1D, ALPH2D, CNACA, CL1CA, CLMCA, CLMCD
    9 FORMAT (7F10.3)
      END

      SUBROUTINE FLT(X,Y,C)
      PI = 3.1415926536
      Z = ATANF (X)
      IF (B) 4,4,5
    1 Y1 = 0.5 * SINF(2. * (Z + Y)) ---------------------------------(13')
      IF (ABSF(Y1 - Y) - 1.E-4) 3,3,2
    2 Y = (Y1 + Y)/2.
      GO TO 1
    3 Y = (Y1 + Y)/2.
      C = TANF(Z + Y)/Y-------------------------------------------------(14')
      B = Y/(PI/2. - Z)
      RETURN
    4 Y = (PI/2. - Z) * 0.48897-----------------------------------------(24')
      GO TO 1
    5 Y = (PI/2. - Z) * B
      GO TO 1
      END

      FUNCTION ALP(X,Y)
      PI = 3.1415926536
      IF (A) 6, 6, 7
    1 ZY = Z + Y
      Z1 = ( X - 1.)/(X *TANF(ZY))--------------------------------------(17')
      IF (ABSF(Z1 - Z) -1.E - 4) 3, 3, 2
    2 Z = Z + (Z1 - Z) * 0.25
      GOTO 1
    3 Z = Z + (Z1 -Z) * 0.25
      IF (Z - PI/2.) 5, 5, 4
    4 Z = PI/2.
    5 ALP = Z
      IF (ABSF(Y) - .01) 9, 9, 8
    6 Z = 0.86 - Y * 0.64-----------------------------------------------(25')
      GO TO 1
    7 Z = 0.86 - Y *A
      GO TO 1
    8 A = (Z - 0.86)/(-Y)
      RETURN
    9 A = 0.64
      END
```

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13. ABSTRACT


The dependences of lift and drag coefficients on angle of attack for bisymmetric lifting vehicles are synthesized using two simplifying assumptions: (1) the axial force coefficient is independent of the angle of attack, and (2) the normal force coefficient increases linearly with the angle of attack. Good agreement with experimental data is found for the value of the angle of attack at which the maximum L/D occurs and the angle of attack for maximum $C_L$ in the hypersonic regime; applicability for subsonic flight is limited to those angles of attack below the inception of flow separation.

Conditions for terminal equilibrium glide and landing for aircraft are derived from the simplified aerodynamic characteristics.

With the linearized normal force, the following useful rules of thumb are obtained for super and hypersonic vehicles with maximum L/D greater than 1.0: (1) the drag at maximum L/D is very closely twice the zero-lift drag, (2) the L/D at maximum $C_L$ is about 0.8, (3) the angle of attack for maximum $C_L$ is about 48 deg, (4) the ratio of the maximum $C_L$ to the $C_L$ at L/D max is approximated by $0.5 + L/D_{max}$, and (5) the ratio of the normal force slope to the maximum lift is closely 2.0 per radian.


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